Effects of Comparing and Discussing Multiple Strategies on Students' Algebra Learning

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Background/Context:

Algebra requires conceptual and procedural knowledge, but recent theories of algebra learning have focused on improving conceptual knowledge (Kieran, 1992). In addition to these knowledge types, procedural flexibility, the ability to know multiple strategies for solving a problem and select the most appropriate strategy, is an important component of successful mathematics learning (Star & Rittle-Johnson, 2008; Woodward et al., 2012). We developed a theory of algebra learning that focused on procedural knowledge and flexibility in addition to conceptual knowledge. We emphasized the importance of multiple strategies, comparison, and discussion. Comparing multiple strategies can improve learning in many domains (Alfieri, Nokes-Malach, & Schunn, 2013; Gentner, Loewenstein, & Thompson, 2003), and comparison may be particularly effective when paired with explanation, as generating explanations improves learning (Aleven & Koedinger, 2002; Hodds, Alcock, & Inglis, 2014; Rittle-Johnson, 2006). Based on evidence from short-term classroom studies (Rittle-Johnson & Star, 2007; Rittle-Johnson, Star, & Durkin, 2009, 2012), we developed a supplemental algebra curriculum comprised of worked example pairs and explanation prompts (Star et al., 2015). With our approach, we hoped to increase comparison and interactive discussions with carefully designed worked example pairs and explanation prompts.

Purpose/Objective/Research Question:

Evidence from short-term studies and a yearlong randomized experiment led to the development of a supplemental algebra curriculum that encourages comparison and discussion of multiple strategies across five topics. We investigated how these materials affected learning compared to typical algebra instruction. We have completed preliminary analyses on the effects of these materials on students' learning of the first topic, solving linear equations.

Setting and Population/Participants/Subjects:

In 2018-2019, 16 Algebra I teachers across four schools in Massachusetts and New Hampshire participated in the treatment group. For the business-as-usual control group, 13 teachers across six schools were recruited with the promise of receiving the materials the following year. We attempted to match schools in the treatment and control groups on key demographics when recruiting participants, but the two groups differed in several important ways. The treatment schools had an average of 17% of students receiving free and reduced price lunch (range 6-39), 5% were African American (range 1-16), 6% were Hispanic (range 3-14), and 77% were white (range 50-90). The control schools had an average of 35% of students receiving free and reduced price lunch (range 10-47), 6% were African American (range 4-14), 26% were Hispanic (range 6-45), and 57% were white (range 32-75). The analytic models described below included these variables as covariates to control for group differences.

Some teachers taught multiple sections of Algebra I, resulting in the 16 treatment teachers covering 25 sections with 550 students and the 13 control teachers covering 21 sections with 498 students. The preliminary analyses for the current study used complete cases with the analytic sample including 475 treatment students and 359 control students.

Intervention/Program/Practice:

We provided treatment teachers with worked example pairs and prompts that encouraged comparison and discussion. For each of five units, teachers were provided with 7 to 9 worked example pairs. The worked example pairs were similar to those used in past research and showed the work of two hypothetical students who solved a math problem followed by prompts for explanation (Star et al., 2015; Figure 1). Treatment teachers participated in professional development for one week during the summer and before beginning each of the units during the school year.

Research Design:

The current study involved an experimental, matched delayed treatment design, as described in the Participants section.

Data Collection and Analysis:

We developed assessments to measure conceptual knowledge (e.g., finding a like term), procedural knowledge (e.g., how to solve a linear equation), and procedural flexibility (e.g., selecting the best way to start a problem) (Rittle-Johnson et al., 2009, 2012). Students completed an overall pretest at the beginning of the year and an identical overall posttest at the end of the year. Students completed a shorter unit test before and after each unit. For the solving linear equations unit, there were 5 conceptual knowledge, 5 procedural knowledge, and 6 flexibility items.

Teachers were videotaped 3 times during each unit. These videos were coded for whether teachers used instructional practices emphasized in our implementation model, including comparing strategies, engaging in small group work, and having a whole-class discussion. Treatment teachers usually included all instructional practices from the implementation model but control teachers did not (Table 1). When they were not coded as using an instructional practice (e.g., only 83% of treatment sections received the whole-class discussion code), it was usually because they did not implement the practice as long as requested.

We used multilevel models to investigate the effect of condition on learning of linear equation solving. These analyses nested students within class sections, which were nested within schools. The outcome of interest was unit posttest score and the predictor of interest was assignment to condition. Overall pretest score, unit pretest score, percentage of students at the school receiving free or reduced price lunch, percentage of African American students, and percentage of Hispanic students were included as covariates. Student-level demographics are not yet available but will be incorporated into later analyses.

Findings/Results:

Treatment students had higher posttest scores than control students (Table 2 and Figure 2). This difference was mainly due to treatment students having higher conceptual knowledge and flexibility. The results from the unit test knowledge subscales must be interpreted with caution due to the small number of items in each scale, but they provide important descriptive information of what might be driving condition differences.

Conclusions:

The preliminary results suggest that using our curriculum encouraged teachers to compare multiple strategies, use small groups, and have mathematical discussions much more frequently than would have happened otherwise. These practices likely led to higher posttest scores, particularly for conceptual knowledge and flexibility, compared to business-as-usual instruction. The preliminary findings from the first target unit are promising that encouraging teachers to compare and discuss multiple strategies can significantly increase students' learning compared to learning in traditional algebra classrooms.

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Table 1

General Fidelity Coding by Condition

Code	% of Treatment Sections	% of Control Sections
Exposed students to multiple strategies	100	8
Multiple strategies were presented side-by-side	100	4
Multiple strategies were compared for at least a 1.5-minute continuous block	97	0
Engaged in partner/small group work focused on math content for at least a 1-minute continuous block	90	42
Had a whole-class discussion for at least a 1.5-minute continuous block	83	12

Table 2

Solving Linear Equation Unit Posttest Results by Condition

	Total		Conceptual		Procedural		Flexibility	
	Coefficient	SE	Coefficient	SE	Coefficient	SE	Coefficient	SE
Intercept	3.62^{***}	0.44	1.08^{***}	0.17	1.75^{***}	0.19	0.95^{***}	0.22
Condition	1.50^{*}	0.64	0.49^{*}	0.24	0.20	0.27	0.93^{**}	0.32
Overall pretest	0.11^{**}	0.03	0.06^{***}	0.01	0.04^{*}	0.02	0.10^{***}	0.02
Unit pretest	0.53^{***}	0.04	0.34^{***}	0.04	0.40^{***}	0.04	0.28^{***}	0.05
FRPL	0.06	0.03	0.02^{**}	0.01	0.02	0.01	0.03	0.02
African American	0.11	0.07	0.06^{*}	0.02	0.05	0.03	0.02	0.03
Hispanic	-0.08	0.04	-0.04^{*}	0.02	-0.04^{*}	0.02	-0.02	0.02

* $p < .05$, ** $p < .01$, *** $p < .001$

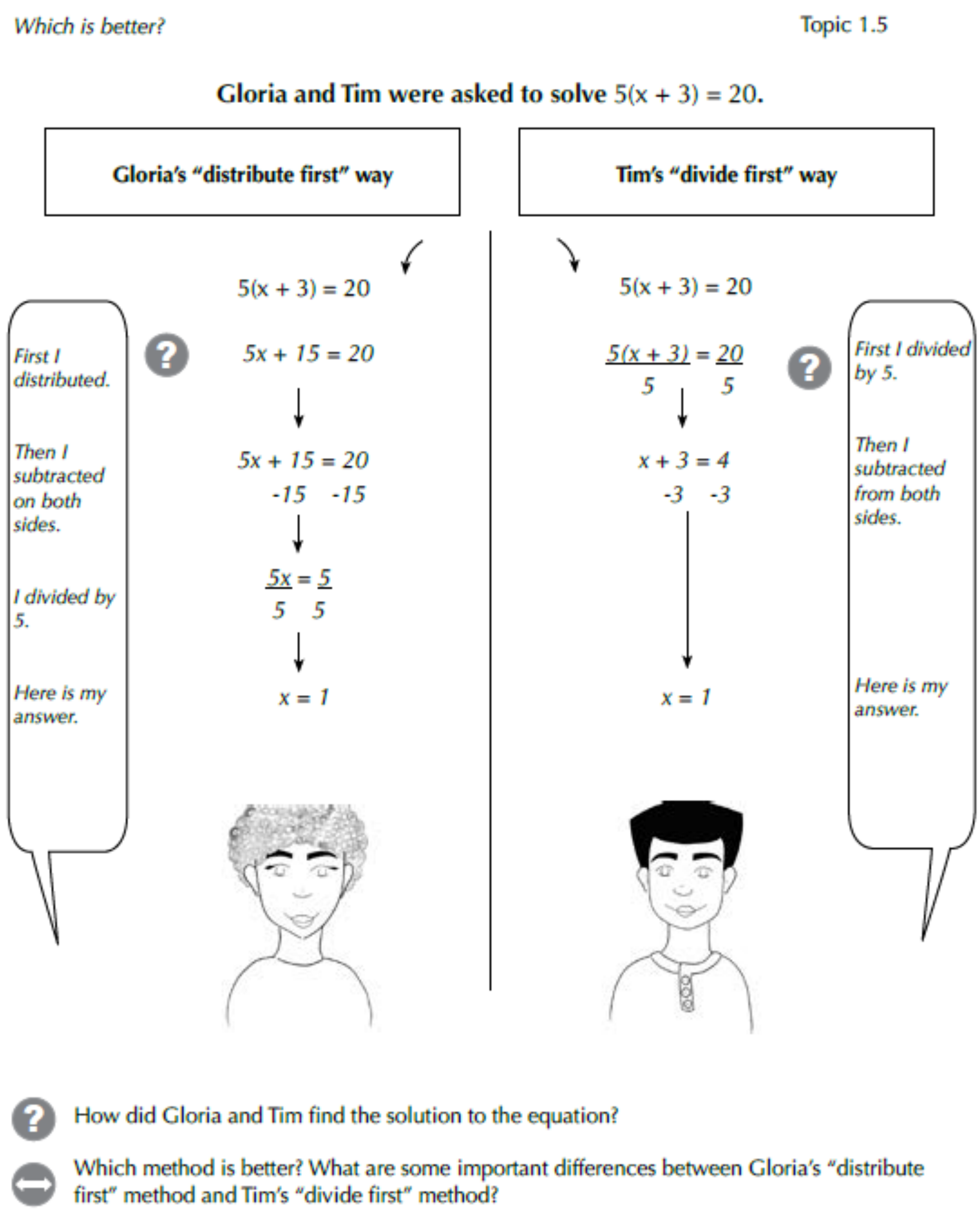

Figure 1. A sample worked example pair and explanation prompts.

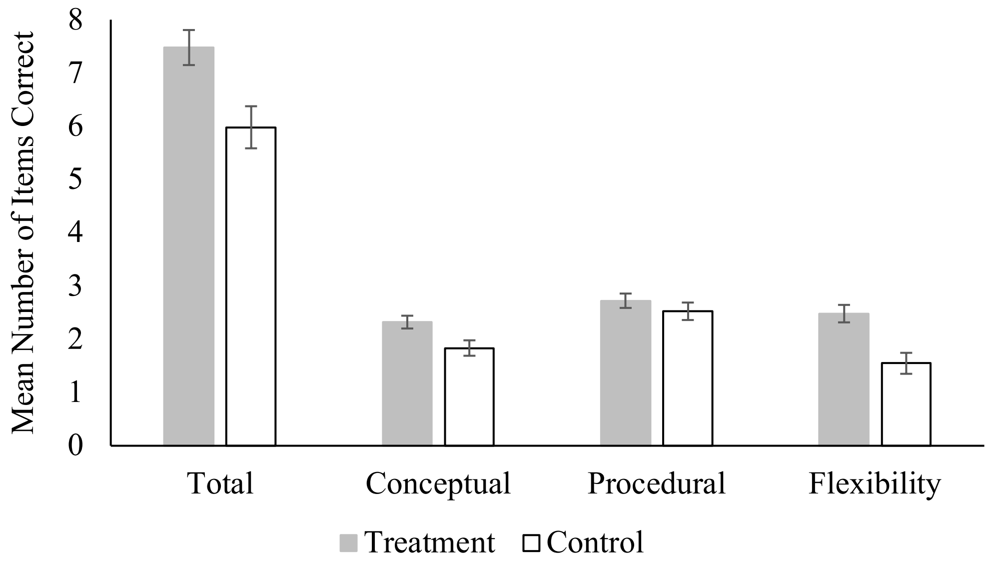

Figure 2. Estimated marginal means of total items (out of 16), conceptual items (out of 5), procedural items (out of 5), and flexibility items (out of 6) correct on unit posttest by condition.