Compare and Discuss to Deepen Algebra Learning

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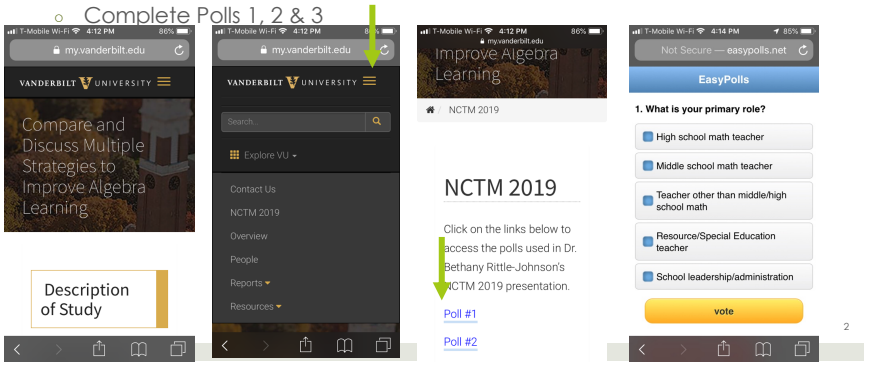
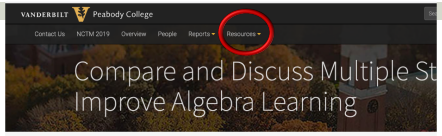
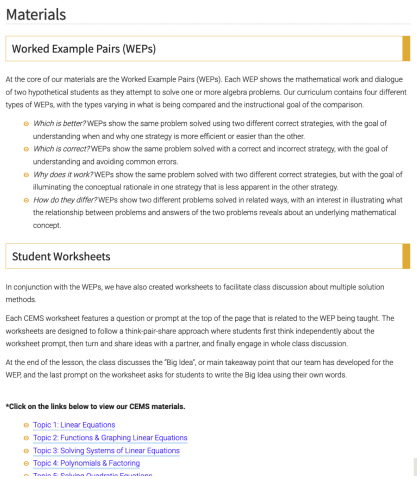


From Resources Tab:
Select Materials
All of our materials can be downloaded





Poll Results

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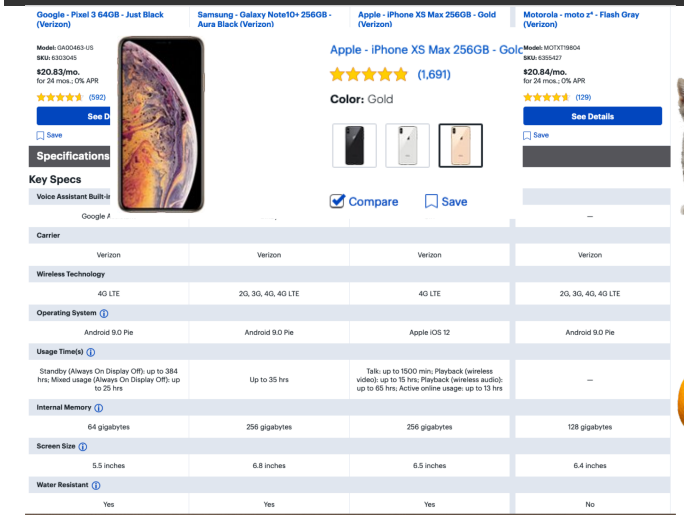
We Often Learn Through Comparison



Babies do too!
Learn categories

Vs.

Learn words

"Comparison is one of the most integral components of human thought"

(Goldstone, Day & Son, 2010, p. 103)

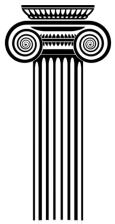
Comparison is a "Best Practice" in Mathematics Instruction

- *Share and compare solution strategies* core to reform pedagogy in many countries (Australian Education Ministers, 2006; Brophy, 1999; Kultusministerkonferenz, 2004; NCTM, 2014; Singapore Ministry of Education, 2006; Treffers, 1991)
- Expert teachers use this approach (Lampert, 1990; Richland, Zur & Holyoak, 2007; Shimizu, 1999)

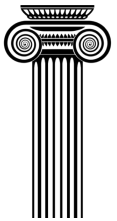
The Essence of Our Approach

Student learning of math, and attitudes toward math, can be improved through the use of:

Comparison



Discussion



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Research

- Over 15 years of research supporting the development and testing of this approach
- Several large grants totaling over $5 million from the US Department of Education and the National Science Foundation
- 30+ publications in academic and teacher journals

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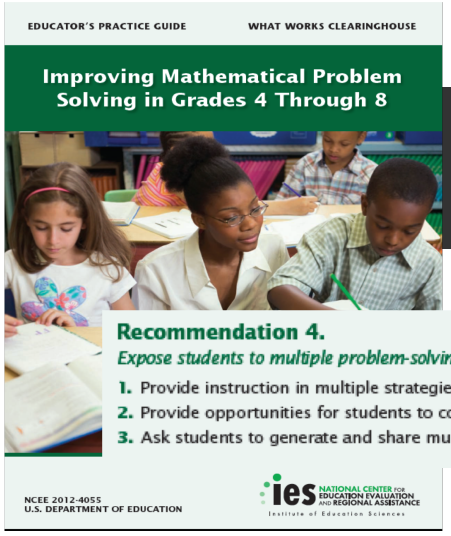
Benefits of Compare & Discuss

- Comparing and discussing multiple strategies improves students'
  - Problem-solving accuracy
  - Flexibility: Knowing multiple strategies and when to use them
  - Understanding of key concepts and strategies



Recommended Teaching Strategy

ies.ed.gov/ncee/wwc/PracticeGuides

Recommended Teaching Strategy

Need to Help Teachers Use Compare & Discuss More Frequently and Effectively

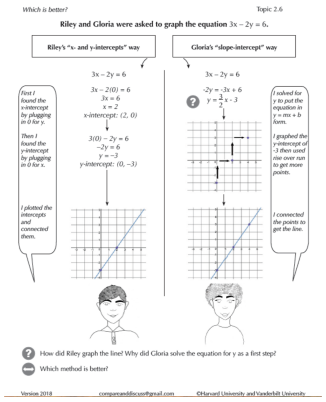
- Comparing strategies rarely done in textbook lessons on **Algebra**
  - Only 1% of examples in a US Algebra I textbook included multiple strategies for solving *the same problem*, and comparison was not supported.

Use of Compare and Discuss in Typical Algebra Classrooms is Infrequent

Instructional Practice	% of Algebra Lessons
Exposed students to **multiple strategies**	20
Multiple strategies were compared for at least a 1.5-minute continuous block	1
Engaged in **partner/small group work** for at least a 1-minute continuous block	29
Had a **whole-class discussion** for at least a 1.5-minute continuous block	6

Compare & Discuss: Worked Example Pairs (WEPs)

- Side-by-side comparison of solved problems
- Shows hypothetical students' work and dialogue explaining process
- Includes discussion questions and prompts



Our Supplemental Compare & Discuss Curriculum for Algebra I

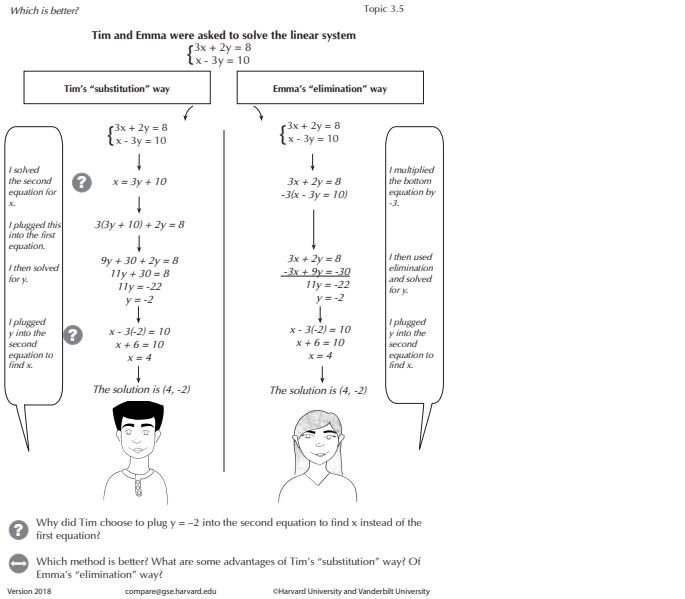
- **Accessible online at**
  - **my.vanderbilt.edu/cems**
    - **Resources tab**
- Materials for each lesson:
  - Teacher Guide for planning
  - Worked-example pair
  - Graphic organizer for student discussion
  - Big Idea
  - (See handout for sample materials)
- **7-9 lessons per topic**

Compare & Discuss Problems

Topic 1: Linear Equations

Video Guiding Questions

- In what ways is this lesson leveraging the power of comparison?
  - How do the **materials** support **comparison**?
  - How does the **teacher** facilitate **comparison**?
- See Handout for Topic 3.5:
  - Worked Example Pair
  - Discussion Connections graphic organizer
  - Summary of Big Idea
- See Handout 2 for space to record your ideas. Will use Think-Pair-Share routine because teachers use this routine with our materials.



Video (Using Topic 3.5 Which is Better?)

Opening to 2:42; *5:36-9:50*

Discussion: Think-Pair-Share

- THINK: 1 minute to finish jotting down your thoughts to the prompts on handout
- PAIR: 5 minutes to pair with another participant to discuss your responses to the questions. (Groups of 3 if needed)
- SHARE: 3-4 groups share out given our limited time

PROMPTS reminder:

- In what ways is this lesson leveraging the power of comparison?
  - How do the **materials** support **comparison**?
  - How does the **teacher** facilitate **comparison**?

Your Observations

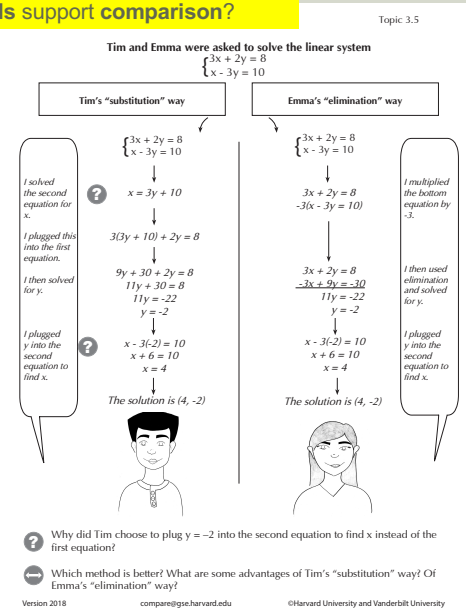
- SHARE: In what ways is this lesson leveraging the power of comparison?
  - How do the **materials** support **comparison**?
    1. x
  - How does the **teacher** facilitate **comparison**?
    1. x

How do the **materials** support **comparison**?

1. Present two different strategies for solving the same problem.

2. Presented as students' solutions to encourage critical reflection.

3. Make both examples visible and clear; present side-by-side.



4. Prompts for students to:
A. Understand each strategy.
B. Compare strategies to identify pros and cons.

How does the teacher facilitate comparison?

1. Prepare to compare: Take time for students to understand each strategy

2. Make Comparisons
- *1. Ask students to explain similarities and differences.*
  - *1.Mark or list them.*
- *2. Push students to reflect on a key point about the comparison.*

Prepare to Compare
- What is the problem asking?
- What is happening in the first method?
- What is happening in the second method?

Make Comparisons
- What are the similarities and differences between the two methods?
  - Which method is better?

Share. After reviewing together, summarize the ideas we agreed on on your handout.

Big Idea

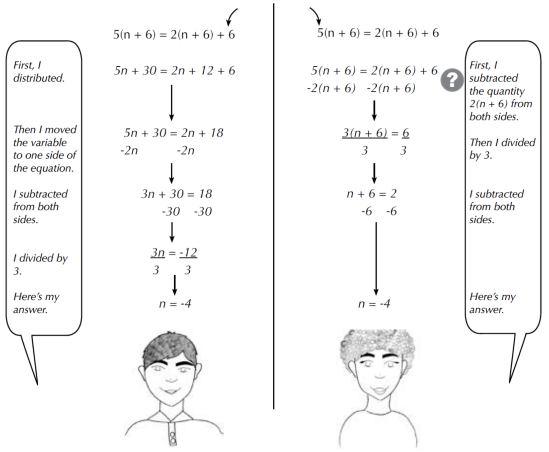
- Multiple techniques make comparing strategies more effective, including side-by-side presentation of the strategies and prompting students to identify similarities and differences and pros and cons of the strategies.

Big Idea. Write what you think is the big idea of this video example and discussion, in your own words.

Video Guiding Questions

- In what ways is this lesson leveraging the power of comparison? What new ideas do you notice?
  - How does the **teacher** facilitate **comparison**?
- See Topic 1.7 worked example pair
- Record ideas on handout 2

Riley and Gloria were asked to solve $5(n + 6) = 2(n + 6) + 6$.



How did Riley and Gloria solve the equation?

Which method is better? What are some important differences between Riley's"distribute first" method and Gloria's "composite variable" method?

Video 2 (Using Topic 1.7 Which is Better?)

Discussion: Think-Pair-Share

- THINK: 1 minute to finish jotting down your thoughts
- PAIR: 3 minutes to pair with another participant to discuss your responses to the question.
- SHARE: 2-3 groups share out given our limited time

PROMPTS:

- In what ways is this lesson leveraging the power of comparison? **What new ideas do you notice?**
  - How does the **teacher** facilitate **comparison**?

How does the teacher facilitate comparison?

2. Make Comparisons

1. Ask students to explain similarities and differences.

1. Use color coding to mark, write notes.

2. Doesn't stop at 1-2 similarities or differences.

2. Push students to reflect on a key point about the comparison.

Make Comparisons

- What are the similarities and differences between the two methods?
  - Which method is better?

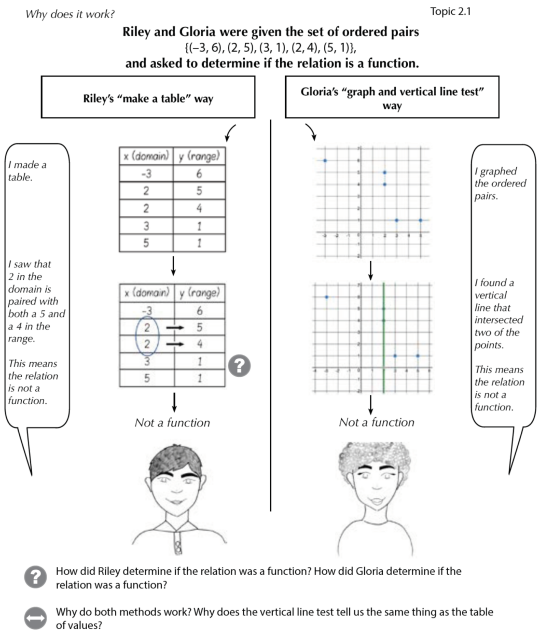
Types of Comparisons

See your handout

- **Type 1: Which is better?**
  - Compare two correct strategies and reflect on *when* one strategy is better than another
  - Examples Topic 3.5 and Topic 1.7 (video)
- Type 2: **Why does it work?**
  - Compare two correct strategies to better understand why the teacher-taught strategy works
  - Example Topic 2.1
- Type 3: **Which is correct?**
  - Compare a correct and incorrect strategy to understand why common mistakes are incorrect and to increase use of correct strategies.
  - Example Topic 4.5

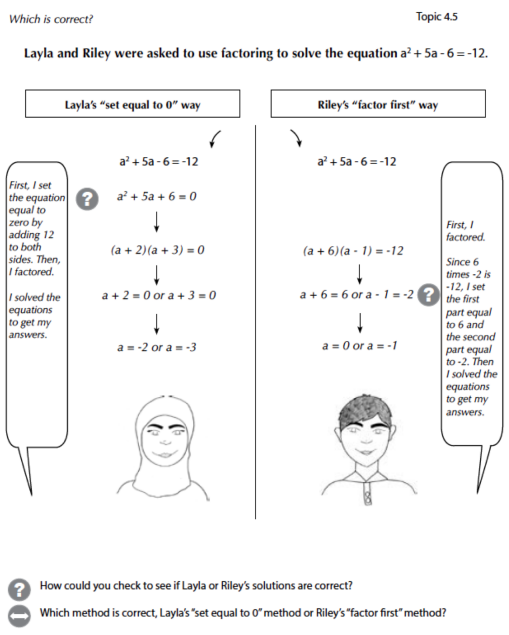
Why Does It Work?

Compare two correct strategies to better understand why the teacher-taught strategy works



Which is correct?

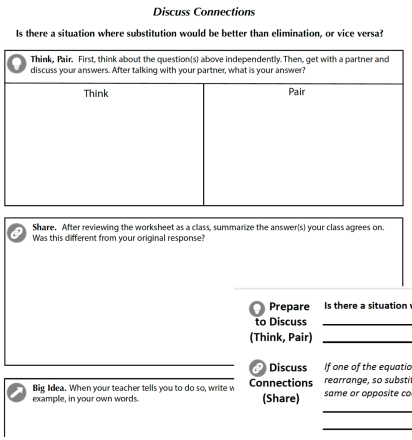
Compare a correct and incorrect strategy to understand why common mistakes are incorrect and to increase use of correct strategies.



How do the **materials** support **discussion**?

Graphic organizer to support
a) **Think – pair – share** routine

b) Students summarizing the big idea in own words



Discuss Connections

Is there a situation where substitution would be better than elimination, or vice versa?

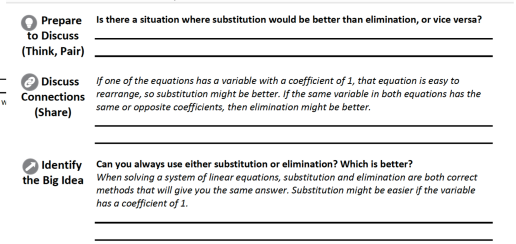
Think, Pair. First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair

Share. After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

Big Idea. When your teacher tells you to do so, write w
example, in your own words.

Teacher Guide includes teacher questions and a potential student answer



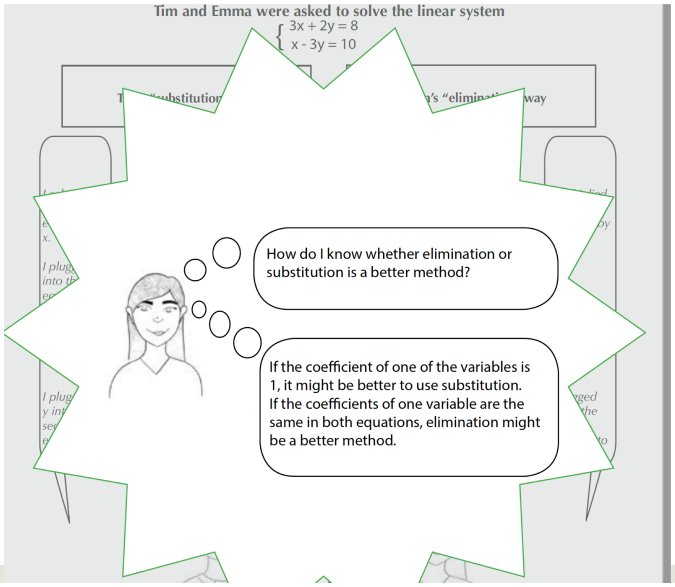
Prepare to Discuss (Think, Pair) **Is there a situation where substitution would be better than elimination, or vice versa?**

Discuss Connections (Share) *If one of the equations has a variable with a coefficient of 1, that equation is easy to rearrange, so substitution might be better. If the same variable in both equations has the same or opposite coefficients, then elimination might be better.*

Identify the Big Idea **Can you always use either substitution or elimination? Which is better?** *When solving a system of linear equations, substitution and elimination are both correct methods that will give you the same answer. Substitution might be easier if the variable has a coefficient of 1.*

How do the **materials** support **discussion**?

Slide summarizing a Big Idea that should emerge from discussion.



Video Guiding Questions

- In what ways is this lesson leveraging the power of discussion?
  - How does the **teacher** facilitate **discussion**?
- See Handouts for Topic 3.5:
  - Worked Example Pair
  - **Discussion Connections graphic organizer**
  - **Summary of Big Idea**

Video of Discussion (Using Topic 3.5 Which is Better?)

Discussion: Think-Pair-Share

- THINK: 1 minute to finish jotting down your thoughts
- PAIR: 5 minutes to pair with another participant to discuss your responses to the questions
- SHARE: 3-4 groups share out given our limited time

PROMPTS:

- In what ways is this lesson leveraging the power of discussion?
  - How does the **teacher** facilitate **discussion**?
    - X

Leveraging the Power of Discussion

How can teachers support **discussion**?

1. During the discussion:
   - Asking open-ended questions (e.g., "Why do you think that's true?")
   - Re-voicing and summarizing contributions
   - Hearing from many voices
   - Holding participants accountable for listening to others: "Do you agree or disagree with Morgan? Why?",

Big Idea

THINK: *On a notecard:*

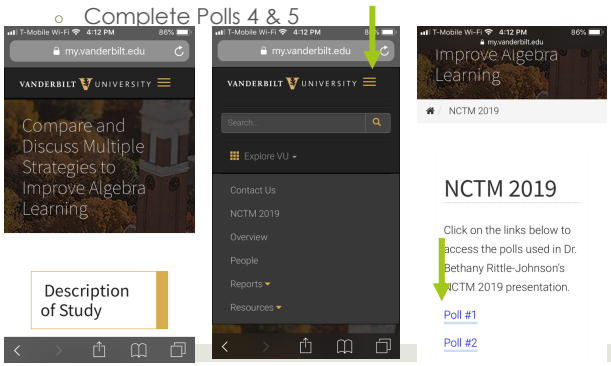
- BIG IDEA: Write what you think is the big idea of this presentation, in your own words.
- USE IT: Write 1-3 things you learned from today that you plan to use in your own instruction.

PAIR: Share with your partner
SHARE: Share with me by dropping off your notecard on front table

Complete Polls 4 & 5 online

Audience Exit Polls 4 & 5

- Go to: my.vanderbilt.edu/cems/
  - Select NCTM 2019 from menu
  - Complete Polls 4 & 5



Poll Results

- http://www.easypolls.net/poll.html?p=5d8d0dc8e4b00d4af40b68ab
- http://www.easypolls.net/poll.html?p=5d8d0dece4b00d4af40b68ac

- Side note: We are hoping to work with Integrated Math I teachers in Metro Nashville Public Schools next year! Let me know if you are an MNPS teacher who might be interested in participating if the project works out.

Acknowledgements

- Slide and Materials available at:
  - **my.vanderbilt.edu/cems**
- E-mail: b.rittle-johnson@vanderbilt.edu
- Funded by grants from the Institute for Education Sciences and the National Science Foundation
  - Opinions expressed are those of the authors only!

Select References

- Rittle-Johnson, B., & Star, J. R. (in press). The power of comparison in learning and instruction: Learning outcomes supported by different types of comparisons. In J. P. Mestre & B.H. Ross (Eds.), *Psychology of Learning and Motivation: Cognition in Education (Vol. 55)*.
- Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561-574.
- Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology, 101*(3), 529-544.
- Rittle-Johnson, B., Star, J. R., & Durkin, K. (2009). The importance of prior knowledge when comparing examples: Influences on conceptual and procedural knowledge of equation solving. *Journal of Educational Psychology, 101*(4), 836-852.
- Rittle-Johnson, B., Star, J., & Durkin, K. (in press). Developing procedural flexibility: When do novices learn from comparing procedures? *British Journal of Educational Psychology*.
- Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology, 102*, 408 – 426.