

How do they differ?

Topic 2.3

### Gloria and Tim were solving the problem

$f(x) = 4x + 1$   
to find  $f(2)$ .

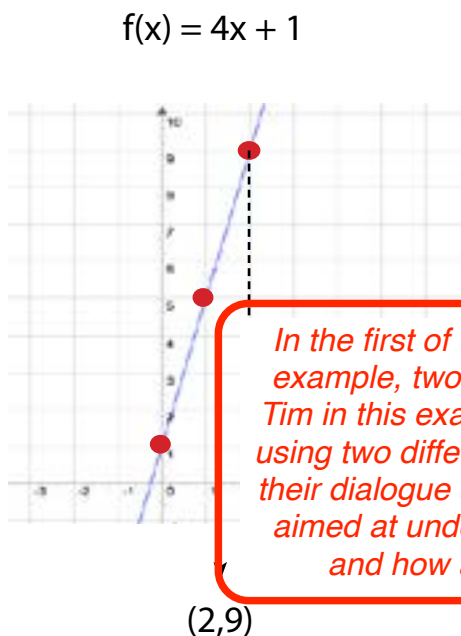
There are four different types of solved examples. In this type ("How do they differ?"), typically the same problem is solved using two different methods.

This number indicates the main topic of the example (in this case, 2 indicates the topic of Functions and Graphing Linear Equations.

The equation is in slope-intercept form, so I know that 4 is the slope and 1 is the y-intercept.

I plotted the y-intercept then continued to plot points using the slope.

I got (2,9) as my answer.



$f(x) = 4x + 1$   
 $f(2) = 4(2) + 1$   
 $f(2) = 8 + 1$

I am solving for the output, and I know 2 is the input.



I got  $f(2) = 9$  as my answer.

In the first of the three pages provided for each example, two hypothetical students (Gloria and Tim in this example) are solving a problem, often using two different methods. Their math work and their dialogue are provided, and the discussion is aimed at understanding what each student did, and how and why these methods work.



These icons accompany questions that might be useful to ask students during the discussion of this example.



How did Gloria know to find 2 on the x-axis instead of the y-axis?




Did Gloria and Tim get the same answer? How do you know?


*In the second page of each example, students engage in a “think-pair-share” routine, about a new but closely related problem.*

### Discuss Connections


Use Gloria’s “graphing” and Tim’s “function notation” ways to find where  $f(x) = 13$ .

 **Think, Pair.** First, think about the question(s) above independently. Then, get with a partner and discuss your answers. After talking with your partner, what is your answer?

Think	Pair
<p><i>Many teachers find it helpful to distribute paper copies of this page and to have students (and their partners) write responses in the boxes.</i></p>	

 **Share.** After reviewing the worksheet as a class, summarize the answer(s) your class agrees on. Was this different from your original response?

*The lesson culminates in a discussion of the “Big Idea,” which is the lesson objective for this example.*

 **Big Idea.** When your teacher tells you to do so, write what you think is the big idea of this example, in your own words.



### Gloria and Tim were solving the problem

$$f(x) = 4x + 1$$

to find  $f(2)$ .

Gloria's "graphing" way

Tim's "function notation" way

*The "Big Idea" page, which is the third page provided with each example, summarizes the lesson objective for this example.*

The equation is in slope-intercept form, so I can find the slope and y-intercept.

I am solving for the output, and I know 2 is the input.

What did I learn from comparing the two ways?



We can use function notation as well as x's and y's to write and graph linear functions. Both  $f(x)$  and  $y$  refer to the output of the function, when  $x$  is the input.

I got (2, 9)

$f(2) = 9$  is my answer

- How did Gloria know to find the x-axis instead of the y-axis?
- Did Gloria and Tim get the same answer? How do you know?

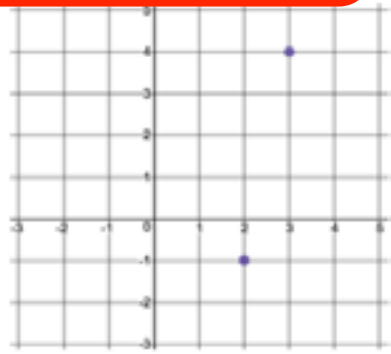
Tim and Emma were asked to find the slope of the line passing through (3, 4) and (2, -1).

In Why does it work? examples, students compare two different methods for solving the same problem, in order to learn more about how and why the methods work.

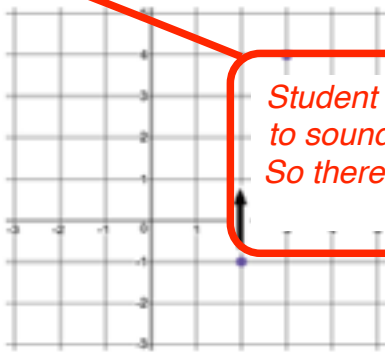
Emma's "formula" way

Each of the students' methods is given a name, which makes it easier for the class to discuss it. The names may also highlight some important features of each method.

First I plotted the two points on a graph.



I started at the bottom and counted up 5 and over 1.



The slope is 5.

$$m = \frac{5}{1} = 5$$



Each of the students' methods is given a name, which makes it easier for the class to discuss it. The names may also highlight some important features of each method.

$$m = \frac{-5}{2 - 3}$$



$$m = \frac{-5}{-1}$$

...ed in the points.

I simplified the numerator and denominator.

$$m = 5$$

The slope is 5.



Student dialogue is intentionally written to sound like what a student might say. So there may be imprecise language or missing steps.

? Tim counted the spaces between the two points, beginning at the point (2, -1). Would Tim have gotten the same answer by starting from the other point, (3, 4)?

↔ Why do both methods work? How does each method show that slope is "rise over run"?

Which is better?

Topic 3.3

Gloria and Tim were asked to solve the linear system

$$\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases}$$

Topic 3 is Systems of Linear Equations.

In Which is better? examples, students compare two different methods for solving a problem, where one of the methods may be better than the other.

Tim's "solve for y" way

I solved the second equation for x.

I substituted this into the first equation.

Then I solved for y.

I plugged this back into the equation I solved for x. Then I found x.

Here is my answer.



$$\begin{aligned} &\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases} \\ &\downarrow \\ &x - 2y = -6 \\ &x = 2y - 6 \\ &\downarrow \\ &4(2y - 6) + 6y = 4 \\ &\downarrow \\ &x = 2y - 6 \\ &x = 2(2) - 6 \\ &x = 4 - 6 \\ &x = -2 \\ &\downarrow \\ &\text{The solution is } (-2, 2) \end{aligned}$$



$$\begin{aligned} &\begin{cases} 4x + 6y = 4 \\ x - 2y = -6 \end{cases} \\ &\downarrow \\ &x - 2y = -6 \\ &-2y = -x - 6 \\ &y = \frac{x}{2} + 3 \\ &\downarrow \\ &4x + 6\left(\frac{x}{2} + 3\right) = 4 \\ &\downarrow \\ &y = \frac{x}{2} + 3 \\ &y = \frac{-2}{2} + 3 \\ &y = -1 + 3 \\ &y = 2 \\ &\downarrow \\ &\text{The solution is } (-2, 2) \end{aligned}$$



I solved the second equation for y.

I substituted this into the first equation.

Then I solved for x.

I plugged this back into the equation I solved for y. Then I found y.

Here is my answer.

For each example, we provide our recommendation about whether it is best used at the beginning, middle, or end of a lesson. This example is labeled "Mid-lesson," as it may be useful in deepening the content of a lesson or even serving as the centerpiece of a lesson. "Beginning" examples are good for introducing new content, while "End" lessons might be good for review or closure.

A discussion using these examples begins by understanding each of the compared examples - by preparing to compare, as indicated by questions with this symbol.



Why did Gloria choose to solve the second equation for the x variable? Why did Tim choose to solve the second equation for the y variable?



Which method do you think

This symbol indicates questions that are best used in a subsequent phase of the discussion, where students are comparing and contrasting the two methods provided in the example.

Which is correct?

Topic 4.3

Riley and Gloria were asked to solve  $8x^2 - 24x = 0$ .

Topic 4 is Polynomials and Factoring.

In Which is correct? examples, students compare two different methods for solving a problem, where one of the methods is correct and the other is not.

Gloria's "divide by x" way

First, I factored out the 8x.

Then, I set 8x and (x - 3) equal to 0 and solved.

Here are my answers.

$$8x^2 - 24x = 0$$

$$8x(x - 3) = 0$$



$$8x = 0 \text{ or } (x - 3) = 0$$



$$x = 0 \text{ or } x = 3$$



$$8x^2 - 24x = 0$$

$$8x^2 - 24x = 0$$
$$+24x \quad +24x$$
$$8x^2 = 24x$$



$$\frac{8x^2}{8x} = \frac{24x}{8x}$$



$$x = 3$$



First, I added 24x to both sides.

Then, I divided by 8x on both sides.

Here is my answer.



If you make your own examples, or if you have feedback or suggestions about any of our examples, we'd love to hear from you!



Why did Riley factor out the 8x?



What is the same or similar about Riley's "factor first" method and Gloria's "divide by x" method? What is different?